Practical aspects of terrain correction in airborne gravity gradiometry

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Summary

We study the practical aspects of performing terrain corrections in airborne/seaborne gravity gradient surveys in terms of necessary spatial extents and required resolution of digital elevation models. We develop a new and practical method for performing terrain corrections through an extension of Parker's 1972 method for calculating potential anomalies in the Fourier domain. We then study and present a discussion of the parameters necessary for an accurate and efficient terrain correction.

Introduction

Gravity gradient data are currently enjoying a boost in popularity as their usefulness in both structural and stratigraphic petroleum exploration is re-discovered (i.e. Pawlowski, 1998) and as their use in mineral exploration is rapidly expanding. Gravity gradiometry's ability to reject common-mode noise, directly measure quantities that must be calculated from potentially noisy data with standard gravity surveys, and its ability to reliably record data from a moving platform assure its place in resource exploration. However, in most applications, a number of the errors and features of gravity gradiometry surveys have not been studied extensively, especially during processing, leading to misinterpretation of the final data.

Though gradiometry is more resistant to many forms of noise than standard vertical-component gravity surveys, reductions and filters still need to be applied. Gradiometry is sensitive to non-linear acceleration of the observation platform (Gupta and Ramani, 1982), resulting in high-frequency contamination of the recorded signal superimposed on the lower-frequency response of non-linear gravity gradients. This high-frequency noise can overwhelm the desired signal, so a low-pass filter is invariably applied to reduce noise (Lee, 2006). The client rarely receives the filter parameters or unfiltered data as a deliverable from the acquisition company, so the effects of these filters are often ignored, leading to introduced error during the terrain correction process.

Often, terrain corrections will have a set radius over which the terrain effect is calculated (i.e. 20 kilometers). However, this value may be more or less than required for a given accuracy depending on the relief (or frequency content) of the terrain to be modeled. It is commonly understood that high resolution terrain data are required to carry out the terrain correction; however, the exact requirement is rarely discussed in the literature. With a better understanding of resolution and spatial requirements, terrain corrections can proceed more quickly and accurately—saving time and money.

To address these considerations, we introduce an extension of Parker's (1972) formula for the Fourier-domain calculation of potential anomalies to create a rapid and robust terrain correction algorithm. We then apply this algorithm to synthetic and actual terrain to understand practical requirements in terrain correction for gravity gradiometry surveys. Specifically we wish to define:

- Required spatial extents of DEM,
- Effect of DEM resolution before and after acquisition filtering, and
- Practical considerations of the Fourier-domain approach.

Optimizing the terrain correction will be an invaluable step in future gradiometry processing for rapid and accurate interpretation and inversion.

Terrain Correction Algorithm

The forward model calculations of a gravity gradient anomaly are computationally efficient in the Fourier domain. In his 1972 paper, R. L. Parker simply and elegantly derived a single expression for any potential field anomaly caused by an uneven layer of material from a planar observation surface with variable density:

$$\tilde{F}[\Delta g] = 2\pi G e^{-\omega_r z_0} \sum_{n=1}^{m} \frac{\omega_r^{n-1}}{n!} \tilde{F}[\rho(\vec{r})\{h^n(\vec{r}) - g^n(\vec{r})\}] \quad (1)$$

where $\tilde{F}$ denotes the Fourier transform, G is the gravitational constant, ω_r is the radial wavenumber vector, ρ is the density (as a function of position), h and g are the top and bottom surfaces of the layer, respectively, and m is some constant required for the summation to converge (for our purposes, m=10 was more than sufficient for convergence). We have altered the original formulation slightly to follow a left-hand coordinate system (x corresponds to easting, y corresponds to northing, and z is positive down).

To generalize this result for the gravity gradient forward model, we of course take the gradient of this function.

$$\tilde{T} = \tilde{F}[\nabla\nabla U] = \tilde{F}[\nabla(\Delta g)] \quad (2)$$

In the Fourier domain, this is accomplished by simply multiplying equation (1) by the appropriate operator. Equations (3a) through (3f) show the explicit definitions of each component in terms of equation (1).

$$\tilde{T}_{xx} = \frac{-\omega_x^2}{\omega_r}\tilde{F}[\Delta g] \quad (3a)$$

$$\tilde{T}_{xy} = \frac{-\omega_x \omega_y}{\omega_r}\tilde{F}[\Delta g] \quad (3b)$$

$$\tilde{T}_{xz} = i\omega_x \tilde{F}[\Delta g] \quad (3c)$$

$$\tilde{T}_{yy} = \frac{-\omega_y^2}{\omega_r}\tilde{F}[\Delta g] \quad (3d)$$

$$\tilde{T}_{yz} = i\omega_y \tilde{F}[\Delta g] \quad (3e)$$

$$\tilde{T}_{zz} = \omega_r \tilde{F}[\Delta g] \quad (3f)$$

To construct an accurate model, we must define the observation region as a subset of the entire model space. Within the observation region, terrain varies as recorded, while outside this subspace the thickness smoothly approaches zero to avoid Gibbs phenomena. When not using a mixed-radix Fourier transform, we must also pad the model space to a power of two.

Required spatial extents

The required extent of the model space to accurately represent the field produced from a source is a function of the lowest frequency component of that field. This is in turn a function of the strength (topographic relief) of the source as well as the perpendicular distance from the furthest point of the source from the observation plane. This clearly has no closed-form solution for every distribution of topographies and densities, so we model all terrain (mountains and valleys) as sums of Gaussians with parameters μ, σ^2, and ρ. By normalizing the height and density of the Gaussian we can empirically derive the minimum required spatial extent through an RMS comparison between differently padded models and the "true" model (a model with very large padding). The result is the absolute minimum requirement for padding in the space domain (in units of distance) from any topographic feature based on its height to account for both the signal due to terrain as well as Fourier effects.

Our first synthetic model consisted of a Gaussian hill with upper boundary 'T' given by:

$$T(x, y) = 300 - 300\exp\left[-\left(\frac{x^2}{300^2} + \frac{y^2}{300^2}\right)\right] \quad (3)$$

and a lower boundary given by a plane at zero meters. The observation plane was fifty meters higher at 350 meters. This represents an average hill (with a 45 degree slope at maximum) that one can expect in normal circumstances with a height of 300 meters. The model occupied 128 by 128 cells.

We forward modeled the gradient response directly, then with additional padding cells at powers of 2: 256, 512, and 1024, using the 1024 as the "true" response. The RMS differences (error) between each of the forward-modeled components and the "true" response components followed an exponential decay with additional padding (Figure 1).

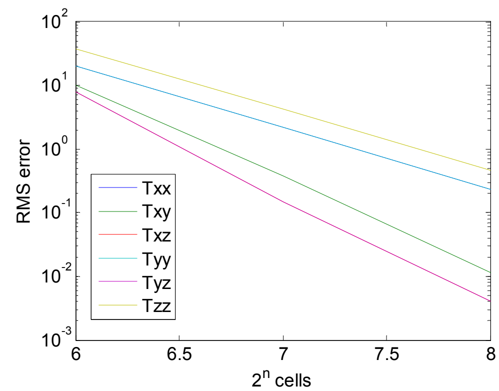

Figure 1. RMS error as a function of padding cells for the synthetic example. Txx and Tyy, as well as Txz and Tyz are identical due to symmetry.

We repeated the experiment with an elliptical Gaussian surface and compared the results with a finite element space-domain calculation for two-layer models to verify the results. Again, the calculated RMS error showed an exponential decay.

We can accurately determine the minimum required spatial extent for a terrain correction based on this approach. In order for the Fourier-domain calculation to yield results accurate to a desired degree of accuracy, we can fit a regression to each component's RMS error to produce empirical formulae for padding cells required as a function of terrain relief with density 2.67 g/cc. Table 1 shows a summary of the calculations.

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Table 1. Factors for padding radii from center of ideal Gaussian topography for each gradient component to achieve 1 Eötvos RMS error or better, where *h* is the height of the terrain with density 2.67 g/cc.

Component	Radius
Txx	6.17 * h
Txy	4.30 * h
Txz	3.83 * h
Tyy	6.17 * h
Tyz	3.83 * h
Tzz	8.49 * h

Effect of terrain resolution

When a low-pass filter is applied to gravity gradiometry data either by instrumentation or by post-acquisition processing, the filter is almost invariably applied in the direction of the flight lines. In addition, because most of the filters' exact properties are proprietary to the acquisition companies (Lee, 2006), exact parameters are unavailable. Therefore, the effect of filters on resolution requirements for terrain correction is unclear.

To study this effect, we processed a set of high resolution LiDAR data (Figure 2) at different effective resolutions both with and without a representative acquisition filter.

Lane (2004) showed that the remaining frequency content of gravity gradiometry data after filtering is consistent with that of a Butterworth filter of varying length. We therefore used a 200 meter length, fourth-order Butterworth filter applied only along the "flight" direction (x, or easting) to simulate the acquisition filter (Figure 3a).

Clearly, most of the high-frequency signal due to the terrain is obliterated, and the filter footprint of striping has changed the relative frequency content between the northing and easting directions. While the RMS difference between the two datasets is small, local distortions cause differences of the same magnitude as the original data (Figure 3b). Up to 73% of the amplitude of the large anomalies has been lost.

Since the higher frequency content signal has been removed, the terrain correction will need to be filtered in the same manner to avoid adding in the high-frequency terrain content that was removed during initial filtering.

Change in resolution can have an equivalently dramatic effect on local features (Figure 4). Changing the resolution from 1 to 30 meters introduced over 400 Eötvos error locally, while the RMS error remained below 1 Eötvos. It is important to note that when studying the adequacy of a particular resolution in terrain correction, RMS differences between maps are a poor measure of validity.

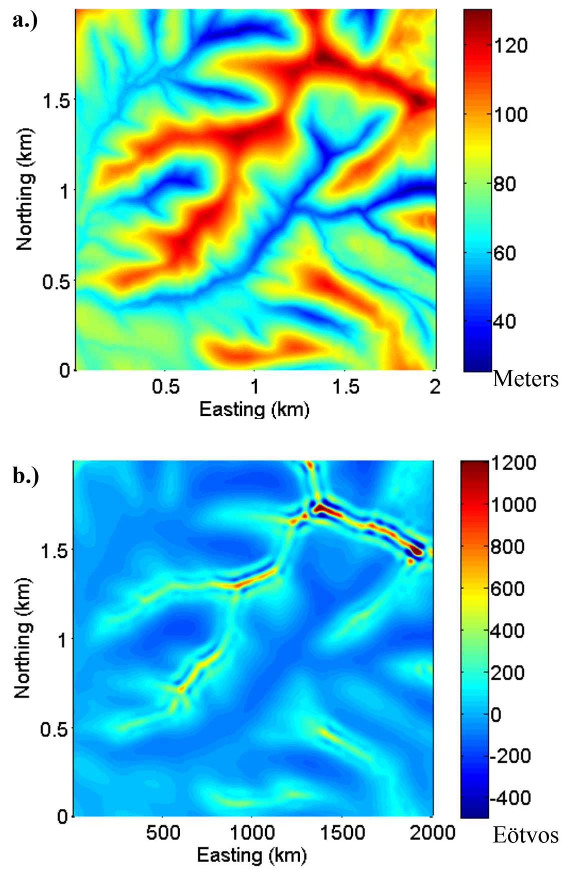

Figure 2. a.) Observation region of terrain used for calculation of resolution requirements. b) Calculated Tzz component of the gravity gradient response due to the high-resolution terrain. The upper 800 and lower 100 Eötvos have been removed from the gradient signal for ease of interpretation.

Practical considerations

Though the Fourier-domain method for calculating the gradient tensor can be up to six orders of magnitude faster than the finite element space-domain method, there are significant limitations in both applicability and computational savings.

As constructed, this method is only applicable for gravity gradiometry data where the flight height is entirely above all the terrain to be modeled. Severe numerical instability occurs as the terrain approaches the observation plane; the

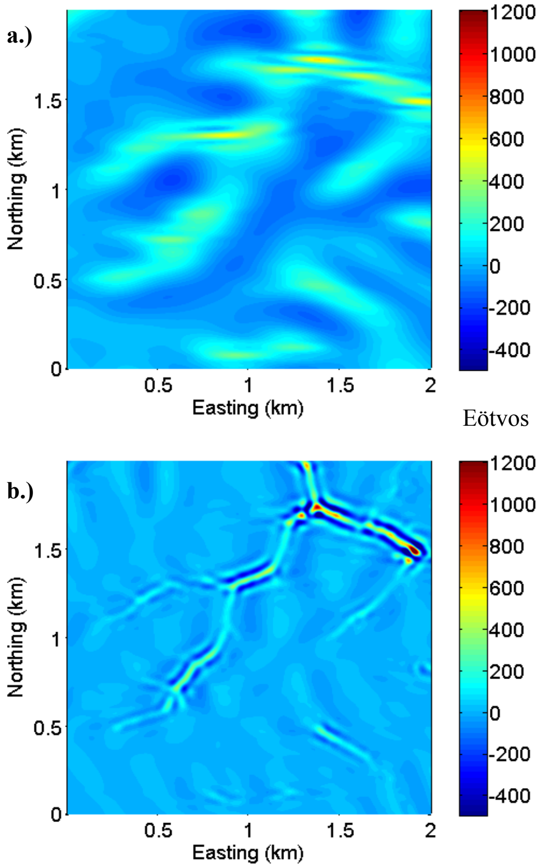

Figure 3. a.) Tzz component of the gravity gradient response due to the high-resolution terrain after acquisition filtering. b.) Difference between the real Tzz component and the filtered Tzz component.

number of summations required approaches infinity. In addition, the observation surface must be planar for this formulation. This limitation is not difficult to meet for most fixed wing surveys, and is ideal for seaborne gravity gradiometry surveys where the survey does not approach land. In Parker's 1995 and 1996 papers, he extends the formulation to allow for terrain violations. Future versions of this algorithm will include these generalizations to allow for non-planar observation surfaces with terrain above and below the surface.

The number of summations that must be performed to reach a nominal misfit in equation (2) is dependent on the amount of relief in the terrain. However, since the summation is only scaled by wavenumber before transformation, the termination condition is simply when the subsequent term in the summation is small relative to the whole, as in any Taylor expansion. Should the required

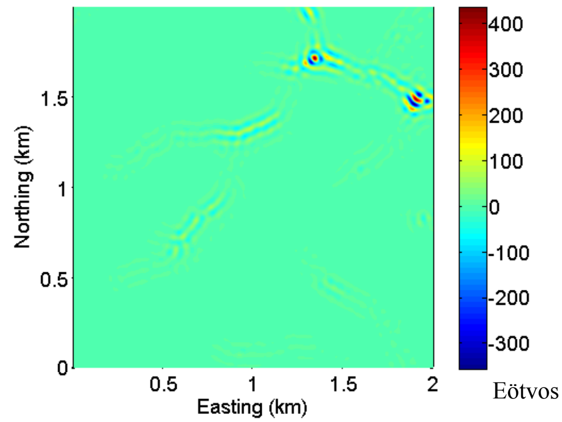

Figure 4. 700 Eötvos error range introduced by using a 30 meter resolution digital elevation model.

number of summations for convergence become large, the computational savings over the space-domain calculation are lost. This problem can be mitigated by subtracting the median elevation to reduce the overall relief that must be modeled. In addition, we work in kilometers to make the quantity $[h^n\text{-}g^n]$ less than zero. When these criteria are followed, we found that ten summations are more than adequate for proper convergence in most cases.

Conclusions

We have shown that oft-overlooked properties of terrain correction can not only lead to much less efficient terrain corrections, but also introduce severe error in the final interpretation.

Ignoring the filter effect on the removal of signal due to terrain will lead to a final processed dataset with high-frequency content due only to the terrain—the very response that was desired to be removed. Incorporating the filter into the terrain correction is the most effective way to mitigate this problem.

We provide analytic solutions for the necessary radii of inclusion in terrain corrections for standardized error. These expressions allow for the reduction of the size of the DEM stored in memory during the calculation, leading to a more efficient correction.

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