Efficient terrain correction in airborne and seaborne gravity gradiometry surveys

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SUMMARY

Proper terrain correction in airborne and seaborne gravity gradiometry surveys requires detailed digital elevation models (DEMs). However, due to standard low-pass filter techniques applied during acquisition, the required resolution of the DEMs is altered in commercial surveys.

We quantify the ideal resolution for an example dataset with a new and practical method for performing terrain corrections based on Parker's 1972 Fourier domain calculation. In the same vein, we quantify the required spatial extents of DEMs for nominal cases. With these parameters quantified, we can optimize the terrain correction to improve efficiency in both gravity gradiometry forward modeling as well as terrain corrections.

In an ideal case, we find that the required spatial extent of the DEM is roughly nine times the greatest relief in the terrain outside the survey area. For the specific case, we find that 20 meter resolution is suitable.

Key words: gravity gradiometry, terrain correction

INTRODUCTION

Gravity gradient data are currently enjoying a boost in popularity as their usefulness in both structural and stratigraphic petroleum exploration is re-discovered (i.e. Pawlowski, 1998) and as their use in mineral exploration is rapidly expanding. Gravity gradiometry's ability to reject common-mode noise, directly measure quantities that must be calculated from potentially noisy data with standard gravity surveys, and its ability to reliably record data from a moving platform assure its place in resource exploration. However, in most applications, a number of the errors and features of gravity gradiometry surveys have not been studied extensively, especially during processing, leading to misinterpretation of the final data.

Though gradiometry is more resistant to many forms of noise than standard vertical-component gravity surveys, reductions and filters still need to be applied. Gradiometry is sensitive to non-linear acceleration of the observation platform (Gupta and Ramani, 1982), resulting in high-frequency contamination of the recorded signal superimposed on the lower-frequency response of non-linear gravity gradients. This high-frequency noise can overwhelm the desired signal, so a low-pass filter is invariably applied to reduce noise (Lee, 2006). The client rarely receives the filter parameters or unfiltered data as a deliverable from the acquisition company, so the effects of these filters are often ignored, leading to introduced error during the terrain correction process.

Often, terrain corrections will have a set radius over which the terrain effect is calculated (i.e. 20 kilometers). However, this value may be more or less than required for a given accuracy depending on the relief (or frequency content) of the terrain to be modeled. It is commonly understood that high resolution terrain data are required to carry out the terrain correction; however, the exact requirement is rarely discussed in the literature. With a better understanding of resolution and spatial requirements, terrain corrections can proceed more quickly and accurately—saving time and money.

To address these considerations, we introduce an extension of Parker's (1972) formula for the Fourier-domain calculation of potential anomalies to create a rapid and robust terrain correction algorithm. We then apply this algorithm to synthetic and actual terrain to understand practical requirements in terrain correction for gravity gradiometry surveys. Specifically we wish to define:

- Required spatial extents of the digital elevation models,
- Effect of DEM resolution before and after acquisition filtering, and
- Practical considerations of the Fourier-domain approach.

Optimizing the terrain correction will be a valuable step in future gradiometry processing for rapid and accurate interpretation and inversion.

TERRAIN CORRECTION ALGORITHM

The forward model calculations of a gravity gradient anomaly are computationally efficient in the Fourier domain. In his 1972 paper, R. L. Parker simply and elegantly derived a single expression for any potential field anomaly caused by an uneven layer of material from a planar observation surface with variable density:

$$\tilde{F}[\Delta g] = 2\pi G e^{-\omega_r z_0} \sum_{n=1}^{m} \frac{\omega_r^{n-1}}{n!} \tilde{F}[\rho(\vec{r})\{h^n(\vec{r}) - g^n(\vec{r})\}] \quad (1)$$

where $\tilde{F}$ denotes the Fourier transform, G is the gravitational constant, ω_r is the radial wavenumber vector, ρ is the density (as a function of position), h and g are the top and bottom surfaces of the layer, respectively, and m is some constant required for the summation to converge (for our purposes, m=10 was more than sufficient for convergence). We have altered the original formulation slightly to follow a left-hand coordinate system (x corresponds to easting, y corresponds to northing, and z is positive down).

To generalize this result for the gravity gradient forward model, we of course take the gradient of this function.

$$\tilde{T} = \tilde{F}[\nabla\nabla U] = \tilde{F}[\nabla(\Delta g)] \quad (2)$$

In the Fourier domain, this is accomplished by simply multiplying equation (1) by the appropriate operator. Equations (3a) through (3f) show the explicit definitions of each component in terms of equation (1).

$$\tilde{T}_{xx} = \frac{-\omega_x^2}{\omega_r}\tilde{F}[\Delta g] \quad (3a)$$

$$\tilde{T}_{xy} = \frac{-\omega_x \omega_y}{\omega_r}\tilde{F}[\Delta g] \quad (3b)$$

$$\tilde{T}_{xz} = i\omega_x \tilde{F}[\Delta g] \quad (3c)$$

$$\tilde{T}_{yy} = \frac{-\omega_y^2}{\omega_r}\tilde{F}[\Delta g] \quad (3d)$$

$$\tilde{T}_{yz} = i\omega_y \tilde{F}[\Delta g] \quad (3e)$$

$$\tilde{T}_{zz} = \omega_r \tilde{F}[\Delta g] \quad (3f)$$

To construct an accurate model, we must define the observation region as a subset of the entire model space. Within the observation region, terrain varies as recorded, while outside this subspace the thickness smoothly approaches zero to avoid Gibbs phenomena. Since we are not using a mixed-radix Fourier transform, we must also pad the model space to a power of two.

EFFECT OF TERRAIN RESOLUTION

When a low-pass filter is applied to gravity gradiometry data either by instrumentation or by post-acquisition processing, the filter is almost invariably applied in the direction of the flight lines. In addition, because most of the filters' exact properties are proprietary to the acquisition companies (Lee, 2006), exact parameters are unavailable. Therefore, the effect of filters on resolution requirements for terrain correction is unclear.

To study this effect, we processed a set of high resolution LiDAR data at different effective resolutions both with and without a representative acquisition filter to forward model the Tzz component of the gravity gradient (Figure 1).

Lane (2004) showed that the remaining frequency content of gravity gradiometry data after filtering is consistent with that of a Butterworth filter of varying length. We therefore used a 200 meter length, fourth-order Butterworth filter applied only along the "flight" direction (x, or easting) to simulate the acquisition filter (Figure 2a).

Clearly, most of the high-frequency signal due to the terrain is obliterated, and the filter footprint of striping has changed the relative frequency content between the northing and easting directions. While the RMS difference between the two datasets is small, local distortions cause differences of the same magnitude as the original data (Figure 2b). Up to 73% of the amplitude of the large anomalies has been lost.

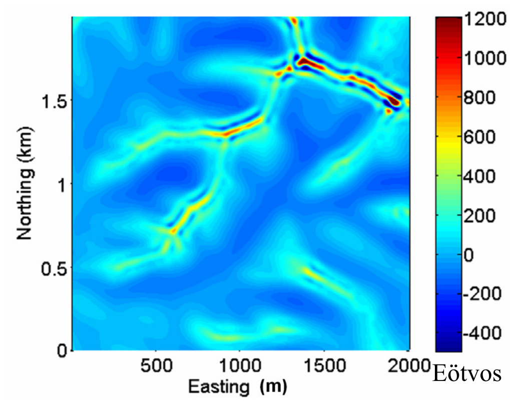

Figure 1. Calculated Tzz component of the gravity gradient response due to the high-resolution terrain. The upper 800 and lower 100 Eötvos have been removed from the gradient signal for ease of interpretation.

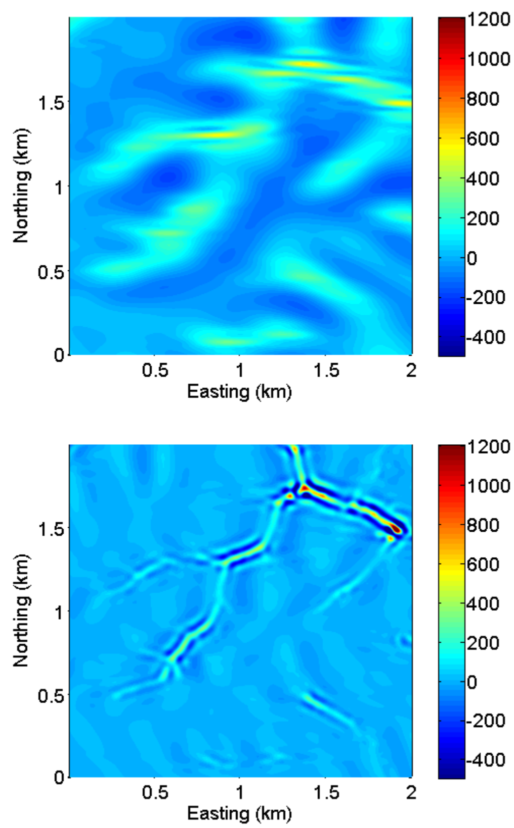

Figure 2. a.) Tzz component of the gravity gradient response due to the high-resolution terrain after acquisition filtering. b.) Difference between the unfiltered Tzz component and the filtered Tzz component.

Since the higher frequency content signal has been removed, the terrain correction will need to be filtered in the same manner to avoid adding in the high-frequency terrain content that was removed during initial filtering.

Change in resolution can have an equivalently dramatic effect on local features. Changing the resolution from 1 to 30 meters introduced over 400 Eötvos error locally, while the RMS error remained below 1 Eötvos. It is important to note that when studying the adequacy of a particular resolution in terrain correction, RMS differences between maps can be a poor measure of validity.

To fully investigate the resolution effect, we calculated the Euclidian and L-∞ norms and errors as a function of decreasing resolution (Figures 3 and 4). The norms of the calculated Tzz components are a related to how much energy is lost with decreasing resolution. While the Euclidian (L-2) norm indicates how much energy is lost in the whole map, the L-∞ norm is a measure of the reduction of the largest anomaly when compared across decreasing resolutions.

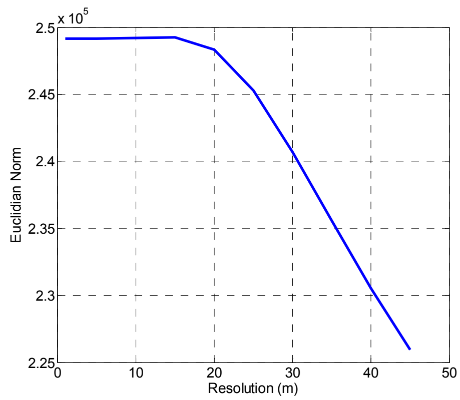

Figure 3. Euclidian norm vs. resolution for the unfiltered Tzz component of the terrain model.

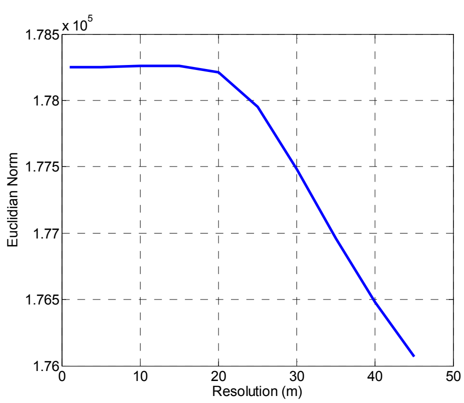

Figure 4. Euclidian norm vs. resolution for the acquisition filtered Tzz component of the terrain model.

Figures 3 and 4 show the non-acquisition filtered data do not improve with a horizontal resolution better than 15 meters, while the post acquisition filtered data do not improve with a resolution better than 20 meters. After 20 meters, the amount of energy lost with decreasing resolution is 10 times slower for the acquisition filtered dataset than for the non-filtered data. Therefore, the gains in accuracy for increasing the resolution for the terrain correction diminish faster when the data have been low-pass filtered. Figures 5 and 6 show the same result through an L-∞ norm in terms of errors. For this case, the optimally accurate terrain correction will be performed at 20-meter horizontal resolution.

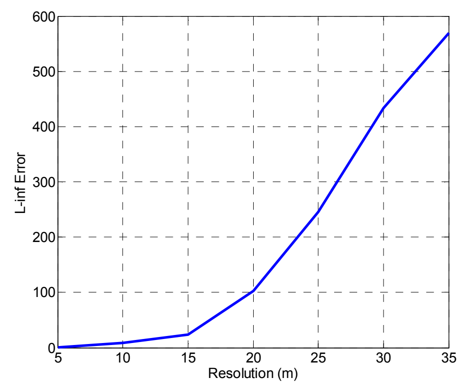

Figure 5. L-∞ error vs. resolution for the unfiltered Tzz component of the terrain model.

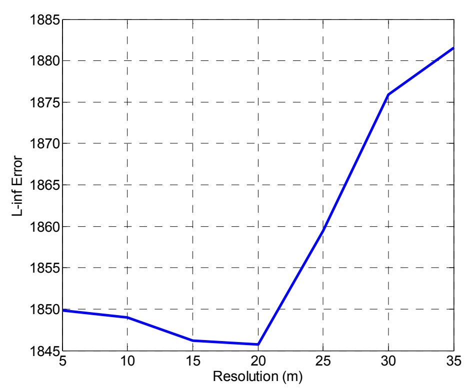

Figure 6. L-∞ error vs. resolution for the acquisition filtered Tzz component of the terrain model.

REQUIRED SPATIAL EXTENTS

The required extent of the model space to accurately represent the field produced from a source is a function of the lowest frequency component of that field. This is, in turn, a function of the strength (topographic relief) of the source as well as the perpendicular distance from the furthest point of the source from the observation plane. This clearly has no closed-form solution for every distribution of topographies and densities, so we model all terrain (mountains and valleys) as sums of

Gaussians with parameters μ, σ^2, and ρ. By normalizing the height and density of the Gaussian we can empirically derive the minimum required spatial extent through an RMS comparison between differently padded models and the "true" model (a model with very large padding). The result is the absolute minimum requirement for padding in the space domain (in units of distance) from any topographic feature based on its height to account for both the signal due to terrain as well as Fourier effects.

Our first synthetic model consisted of a Gaussian hill with upper boundary 'T' given by:

$$T(x,y) = 300 - 300\exp\left[-\left(\frac{x^2}{300^2} + \frac{y^2}{300^2}\right)\right] \quad (4)$$

and a lower boundary given by a plane at zero meters. The observation plane was fifty meters higher at 350 meters. This represents an average hill (with a 45 degree slope at maximum) that one can expect in normal circumstances with a height of 300 meters. The model occupied 128 by 128 cells.

We forward modeled the gradient response directly, then with additional padding cells at powers of 2: 256, 512, and 1024, using the 1024 as the "true" response. The RMS differences (error) between each of the forward-modeled components and the "true" response components followed an exponential decay with additional padding (Figure 7).

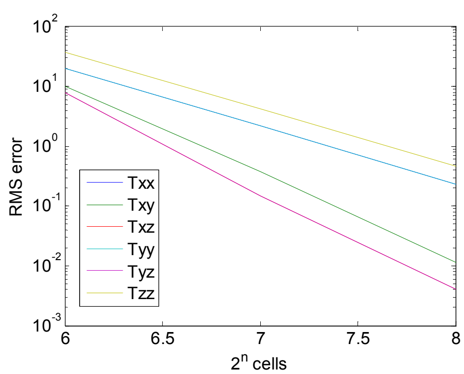

Figure 7. RMS error as a function of padding cells for the synthetic example. Txx and Tyy, as well as Txz and Tyz are identical due to symmetry.

We repeated the experiment with an elliptical Gaussian surface and compared the results with a finite element space-domain calculation for two-layer models to verify the results. Again, the calculated RMS error showed an exponential decay within the observation window.

We can accurately determine the minimum required spatial extent for a terrain correction based on this approach. In order for the Fourier-domain calculation to yield results accurate to a desired degree of accuracy, we can fit a regression to each component's RMS error to produce empirical formulae for padding cells required as a function of terrain relief with density 2.67 g/cc. Table 1 shows a summary of the calculations.

Table 1. Factors for padding radii from center of ideal Gaussian topography to achieve 1 Eötvos RMS error, where *h* is the height of the terrain.

Component	Radius
Txx	6.17 * h
Txy	4.30 * h
Txz	3.83 * h
Tyy	6.17 * h
Tyz	3.83 * h
Tzz	8.49 * h

CONCLUSIONS

We have shown that oft-overlooked properties of terrain correction can not only lead to much less efficient terrain corrections, but also introduce severe error in the final interpretation.

Ignoring the filter effect on the removal of signal due to terrain will lead to a final processed dataset with high-frequency content due only to the terrain—the very response that was desired to be removed. Incorporating the filter into the terrain correction is the most effective way to mitigate this problem.

We provide analytic solutions for the necessary radii of inclusion in terrain corrections for standardized error. These expressions allow for the reduction of the size of the DEM stored in memory during the calculation, leading to a more efficient correction. In our example dataset, we find that 20 meter resolution is the maximum resolution needed for an efficient terrain correction.

ACKNOWLEDGMENTS

We thank Misac Nabighian for his valuable insight. We also thank Jeongmin Lee and Korea Resources Corporation for the funding support, and Sky Research, Inc. for the LiDAR data. This work is also partly supported by the Gravity and Magnetics Research Consortium (GMRC).

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