Hybrid optimization for a binary inverse problem

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Summary

We have developed a hybrid optimization algorithm for inversion of gravity data using a binary formulation. The new algorithm utilizes the Genetic Algorithm (GA) as a global search tool, while implementing Quenched Simulated Annealing (QSA) intermittently for local search. The hybrid has significantly decreased computational cost over GA or Simulated Annealing (SA) alone and has allowed for successful inversion of more realistic gravity problems. We illustrate the algorithm using a large 2½D model derived from the SEG/EAGE 3D salt model, which has a complex background density profile and a pronounced nil zone.

Introduction

When a salt body of uniform density is located at a depth where the sedimentary density is equal to the salt density within a depth interval, a region of salt referred to as a nil zone exists. Because of the zero contribution to surface gravity data from the nil zone, gravity inversion algorithms typically produce poor result near a nil zone. Furthermore, the cancellation of gravity effects due to salt above and below the nil zone renders portions of a salt body invisible to the surface gravity data and difficult to image without incorporating specific information.

To overcome difficulties associated with nil zones, we use a binary formulation (Krahenbuhl and Li, 2002) that enables one to incorporate the density contrast values, a strength of non-linear interface inversion, while retaining the flexibility and linearity of density (cell based) inversion. Because the variable can only take on discrete values, 0 or 1 for sediment or salt respectively, derivative-based minimization techniques are no longer applicable.

There are two obvious techniques for carrying out such discrete-variable minimization problems, namely, genetic algorithm (GA) and simulated annealing (SA). Both are ideally suited for working with binary values. The GA has the advantage of easily incorporating background density information and any previous inversion results simultaneously. It can also easily locate the general vicinity of the solution in the model space. The drawback is that it has a slow convergence and requires a large number of iterations to obtain the final solution. The SA, on the other hand, can zero in on a final solution rapidly but requires a great deal of work to reach this neighborhood.

When combining the two methods, by modifying the GA for course global search and QSA for local search, the resulting hybrid method retains the advantages of both algorithms and has a much higher computational efficiency: it is faster by an order of magnitude than either GA or SA alone. The GA allows for rapid build-up of the larger model features, while QSA rapidly modifies small-scale structure within the model. As a result, the new algorithm enables us to solve realistic problems with a large number of parameters and a complex background density profile.

In the following, we will first review the methodology of the binary formulation for gravity inversion and discuss its solution by genetic algorithm and simulated annealing alone. We will develop a hybrid formulation and illustrate it using a 2½D gravity problem with a large number of unknown parameters, a variable background density profile and a nil zone at depth.

Binary formulation

The difficulty caused by the presence of a nil zone, as discussed in the Introduction, can only be overcome by incorporating prior information to restrict the class of admissible models. We impose the condition that the density contrast must be the discrete values appropriate for the geologic problem. In the simplest form, density contrast is restricted to being either zero or a known value at a given depth. Similar binary approach has been used before. For example, Litman *et al*. (1998) invert for the shape of a scatterer by assuming a constant electrical conductivity value for the background and the scatter, respectively. We adopt the Tikhonov regularization approach and formulate the inversion for the general case of salt imaging at the presence of density reversal by working with discrete density contrasts. The problem then becomes one of minimizing a model objective function subject to allowing model parameters to attain only one of two values at each depth. The objective function, therefore, consists of the weighted sum of a model objective function ϕ_m and data misfit ϕ_d:

$$\begin{aligned} &\text{min.} && \phi = \phi_d(\rho) + \lambda\phi_m(\rho), \\ &\text{subject to} && \rho \in \{0, \Delta\rho(z)\}. \end{aligned} \tag{1}$$

where ϕ_d is formulated as the χ^2 measure of our data misfit, and ϕ_m limits the solution of admisible models to those that are compact and structurally simple.

Our binary formulation is unique in that it incorporates a binary variable τ into the density function of eq.(1) through expected density contrast at depth z :

$$\tau(\vec{r}) \in \{0,1\}. \tag{2}$$

$$\rho(\vec{r}) = \tau\Delta\rho(z) . \tag{3}$$

At a given depth, a value of zero in the model, τ, indicates a zero density contrast (host sediments), while a value of one corresponds to the expected salt density contrast at that depth. The minimization problem is then expressed in $\tau(\vec{r})$ and we can simply work with 0 and 1 for the minimization problem. The actual density contrast value is incorporated into the forward modeling of predicted data during the inversion.

The solution to this problem will be better constrained than formulations that allow continuous values within an upper and lower bound. Although still non-unique, this problem no longer has an infinite number of possible solutions: there are a finite number of cells within the model mesh and only two possible values for each location.

Genetic algorithm and simulated annealing

The Genetic Algorithm (GA) has been the primary tool for development and testing of the binary formulation for gravity inversion. GA is a programming tool designed for solving a variety of optimization problems. It is a stochastic search technique that mimics natural biological evolution by imposing the principle of 'survival of the fittest' on a population of individuals. The main objective is to recombine the individuals, with the better-fit individuals having higher probabilities of reproduction, in order to evolve to better solutions. From an inversion standpoint, the GA attempts to evolve a population of models to a final solution by ranking individuals and combining desired model features at each generation. Detailed information on the components of the GA for binary gravity inversion is available in Krahenbuhl and Li (2002). Additional information on GA may also be found in Goldberg (1989).

There are two primary advantages of the GA for binary inversion. First, the GA is ideally suited for minimization with the binary variable/model τ, composed entirely of zeros and ones. Cells within a geophysical model translate nicely to a string of chromosomes for GA combination. The second advantage is its ability to work with multiple models at one time through the creation of a population, which allows the user to easily incorporate prior information. This information may be in the form of models from previous inversions, or top of salt from pre-stack depth migrated seismic data.

Simulated Annealing (SA) is another global search technique well suited for gravity inversion with binary variable. SA formulation is designed to mimic the process of chemical annealing, where the final energy state of a crystal lattice is determined by its rate of cooling through the melting point. To achieve a lower energy state with highly ordered crystals, the material is usually cooled very slowly. For geophysical inversion, the SA typically starts with a model at random, and calculates the model's energy based on its objective value, eq.(1). Perturbations are then applied to the model and the new objective values are calculated at each iteration. If the new objective value is decreased or remains the same, the model is accepted as a replacement. If the objective value increases, the model is accepted by a thermally controlled probability function often referred to as the Metropolis criterion:

$$P = \exp\left(-\frac{\Delta\phi}{T}\right), \tag{4}$$

where $\Delta\phi$ is the difference between objective values of the new and old models, and T is a temperature control parameter designed to decay (or cool) over time. For more information on SA and the Metropolis cooling function, the reader is referred to Metropolis et al. (1953).

Similar to GA, simulated annealing is well suited for the binary inverse problem. Perturbation to the geophysical model requires a mere binary flip from 1 to 0, or vice versa. Forward calculation of the gravity response is therefore rapid, with the contribution of the flipped cell either added or subtracted based on the new value of 0 or 1. With a temperature controlled cooling function, SA works as a global search technique.

Disadvantages of GA and SA: As described above, GA and SA generate improved solutions to the inverse problem by utilizing information about the objective function directly rather than using derivative information. Because of this, they are among a small list of techniques available for the binary inverse problem. Unfortunately, they are both computationally expensive. The Genetic Algorithm may require thousands of forward modelings at each iteration because it works with a large population of models. Although SA does not work with a large population of models and has a significantly faster forward calculation than GA in binary form, SA requires similar computational cost due to the increased number of iterations over GA without the guidance of derivative information at the early stages. For these reasons, it is a common perception that GA and SA are not feasible techniques for real world geophysical inverse problems with tens (or hundreds) of thousands of parameters.

Hybrid optimization

To decrease computational cost of inversion with the binary formulation, a new technique is required. There are a number of publications on the cost advantages of hybrid optimization for geophysical inversion (e.g. Chunduru et al., 1997). The methods are designed to take advantage of the better features of both global and local search algorithms.

Assuming GA and/or SA work well as a coarse global search technique that can rapidly build large-scale structure within the models, the next step in designing a hybrid optimization algorithm is to choose an appropriate local search technique that is capable of working with the binary formulation. We have opted to use a modified SA, called Quenched Simulated Annealing (QSA), as the local search tool in the hybrid algorithm. QSA in its simplest form is Simulated Annealing described previously, without the Metropolis or any other cooling criteria. Therefore the algorithm only accepts downhill (or lateral) moves.

When combined with a Genetic Algorithm, QSA acts to improve the top fit individual after a prescribed number of generations throughout the inversion by modifying the finer details of the model. Because the highest fit individual in the GA population has a strong influence on the evolution of the entire population, the hybrid between GA and QSA results in faster convergence.

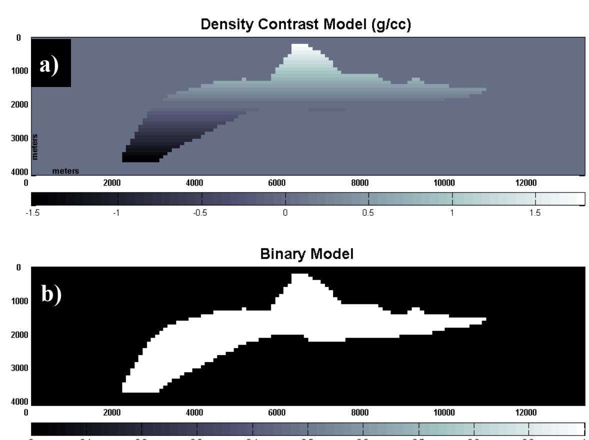

Figure 1. Modified section of SEG/EAGE salt model for gravity inversion. Top (a) is in density contrast form. Bottom (b) is in binary form.

Numerical example:

SEG/EAGE 3D salt model: To illustrate the performance of the hybrid algorithm, we test it on an example satisfying the following three criteria: complex density profile, irregular shape of anomaly source, and a large number of parameters. The 2½-D section is modified from a 2D section through the SEG/EAGE salt model of Aminzadeh *et al*. (1997). A nil zone is present around the depth of 2000 m (Figure 1).

There are 5670 cells in the model and 41 data are collected in profile above the model. To carry out inversion, we incorporate the top of salt as prior information, along with an expected density contrast function. The algorithm then solves for the shape of the lower portion of the salt body.

Result: Figure 2 shows the convergence curve of pure GA minimization and that of the GA/QSA hybrid technique. For GA, there is little difference in objective values between the top fit individual and the average of the population with 1000 individuals at each generation. As a result, convergence is reached after 500 generations and a total CPU time of 5 hours on a 2.4-GHz PC.

The lower panel in Figure 2 illustrates convergence of the hybrid method. It reduces the need for a large GA population with large genetic diversity, as well as reduces the number of generations required for convergence. When QSA is incorporated every five generations of the GA, a population of 100 reaches convergence after 50 generations and a total CPU time of approximately 4 minutes on the same PC. The observed impact on model evolution is that the GA dominates in the build-up of the larger model features. QSA, on the other hand, modifies the top-ranking individual by rapidly developing the finer details in the model, and therefore increasing the evolutionary 'jumps' in the larger hybrid algorithm. This result is also apparent in the performance chart of Figure 2(b) with evolutionary jumps occurring every 5 generations.

Figure 3 shows the inversion results using the GA/QSA hybrid technique by model evolution. After 50 generations, the resulting model is a good representation of the true model. In contrast to continuous variable formulations, inversion results with the binary constraint illustrate the technique's ability to properly image the salt body by filling in the nil zone. The GA/QSA hybrid has likewise reduced computational cost to less than four minutes for this problem, in contrast to five hours for standalone GA or SA.

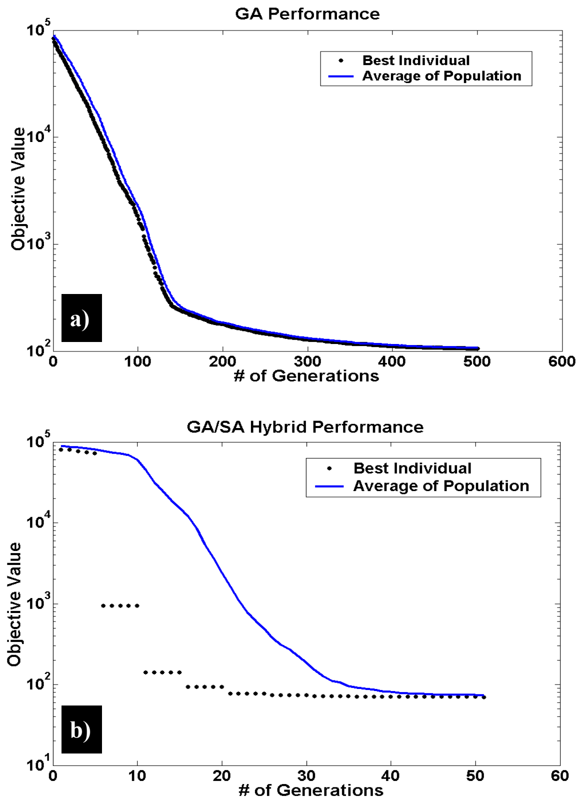

Figure 2. Comparison of performance between stand-along GA (a), and GA/QSA Hybrid (b).

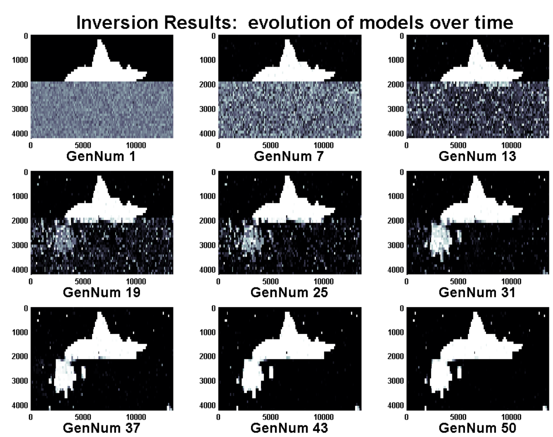

Figure 3. Inversion results for binary formulation with GA/QSA Hybrid algorithm. Results are presented as model evolution over time (average of entire population of models).

Discussion

We have developed a hybrid optimization algorithm for inversion of gravity data using a binary formulation. The binary condition is designed to invert gravity data for subsurface structure that has well-defined density contrasts. The method can overcome the difficulties introduced by nil zones in salt imaging and provides a sharp boundary for the subsurface while maintaining the flexibility of density inversions. The hybrid algorithm, utilizing the Genetic Algorithm and Quenched Simulated Annealing, has significantly decreased computational cost over other derivative-free techniques available for the binary inverse problem. In addition, the hybrid algorithm applied to the binary formulation has successfully solved a 2½D gravity inverse problem with complex background density profile, a large number of parameters, and has filled in the nil zone in salt imaging.

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