Gravity inversion using a binary formulation

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Summary

When a nil zone is present in the subsurface salt structure, it effectively creates an annihilator of density contrast that gives rise to zero gravity response on the surface. As a result, part of the salt structure is invisible to the surface data and inversion algorithms often have difficulties in recovering the salt structure correctly. We develop a binary inversion technique in which the density contrast is restricted to being one of the two possibilities: either zero or the value expected at a given depth. The binary condition places a strong restriction on the admissible models so that the non-uniqueness caused by nil zones can be resolved. In this presentation, we will outline the formulation, discuss the solution strategy, and illustrate it with numerical examples.

Introduction

Inversion methods for imaging salt structure using gravity data can be divided into two general categories. The first are interface inversions. These methods assume a simple topology for the salt body and known density contrast and construct the base of the salt (e.g., Jorgensen and Kisabeth 2000). Methods in the second category are generalized density inversions. These methods construct a density contrast distribution as a function of spatial position and image the base of salt by the transition in recovered density (e.g., Li 2001).

The interface inversion has the advantage that it directly inputs the known density contrast at each depth and provides a direct image of base of salt. However, difficulties may arise from the nonlinear relationship between observations and salt boundaries. In addition, the assumed simple topology of salt creates difficulties when either regional field or small-scale residuals due to shallow sources are not completely removed.

The density inversion has the flexibility of handling multiple anomalies and the solution is easier to obtain because the relationship between observations and the density contrast is linear. However, these methods, as they are currently formulated, are not well suited for the cases where nil zones are present. The reason is the following.

These methods start by discretizing a 3D model into a large number of cuboidal cells, with each cell assuming a constant density value between a lower and upper bound. The solution is obtained by minimizing a model objective function measuring the structural complexity of the density model subject to fitting the data to an appropriate degree. This approach works well when the density contrast is single-signed, i.e., it is either entirely positive or negative. Difficulty arises, however, when the salt is located at a depth such that the sedimentary density is equal to the constant salt density within a depth interval inside the salt.

When this occurs, the density contrast reverses sign as the depth increases and the gravity anomalies due to top and bottom portions of the salt cancel out. Consequently, a portion of the salt body is invisible to the surface gravity data. Inversion allowing continuous density values will in general produce a model that has little resemblance to the true structure. The data are satisfied by intermediate density values.

To overcome difficulties associated with both methods, we propose a binary formulation that enables one to incorporate the density contrast values into the inversion while retaining the flexibility and linearity of the density inversion.

Theory

The difficulty of an annihilator can only be overcome by incorporating prior information to restrict the class of admissible models. We propose to impose the condition that the density contrast must be the discrete values appropriate for the geologic problem. In the simplest form, we restrict the density contrast to being either zero or a known value at a given depth. Staying within this constraint, the problem becomes one of minimizing an objective function subject to allowing model parameters to attain only one of two values at each depth:

$$\begin{aligned} &\text{min.} && \phi = \phi_{\mathrm{d}}(\rho) + \lambda\phi_{\mathrm{m}}(\rho), \\ &\text{subject to} && \rho \in \{0, \Delta\rho(\mathrm{z})\}. \end{aligned} \qquad (1)$$

where ϕ_d is the data misfit function, ϕ_m is the model objective function, and $\Delta\rho(z)$ is the expected density contrast at depth z. For convenience, we introduce a new binary model parameter $\tau(\bar{r}) \in \{0,1\}$ such that

$$\rho(\bar{r}) = \tau\Delta\rho(z)\,. \qquad (2)$$

At a given depth, a value of zero for τ indicates a zero density contrast (host sediments), while a value of one corresponds to the expected salt density contrast at that depth. The minimization problem is then expressed in $\tau(\bar{r})$ and we can simply work with 0 and 1 for the minimization problem. The actual density contrast value is incorporated into the forward modeling of predicted data during the inversion.

The solution to this problem will be much better constrained than formulations that allow continuous values within an upper and lower bound. Although still non-unique, this problem no longer has an infinite number of possible solutions: there is a finite number of cells within the model mesh, and only two possible values for each location. For instance, production of an equivalent source layer at any depth is no longer possible. The binary value of 1, at a specified depth, represents a well-defined density contrast value, either positive or negative with corresponding magnitude. Because of this constraint, a combination of a positive and negative anomaly in gravity data is not reproducible by a source distribution at one depth alone (equivalent source). Whenever an annihilator is present, any geologically unreasonable model that reproduces the data by a combination of density contrasts of intermediate values is automatically eliminated.

The minimization problem defined by eq.(1) has a deceptively simple appearance, but its solution is not trivial. The difficulty lies in the discrete nature of the density contrast. Because the variable can only take on discrete values, derivative-based minimization techniques are no longer applicable. There are several alternative methods for carrying out this minimization. The first, obvious technique is mixed integer programming since our variable to be recovered can only assume a value of either 0 or 1. However, solution of the integer programming problem is both theoretically and numerically complicated.

The second technique involves the use of a controlled random search technique such as genetic algorithms (GA) or simulated annealing (SA). Both methods are ideal for combinatorial minimization, which is the problem we have. The genetic algorithm is well suited for minimizing an objective function with discrete variables, as described in the following sections. In addition, both GA and SA can be implemented with relative ease compared to an integer programming solution. To expedite the research on binary inversion, and to gain basic understanding about the behavior of such approaches, we opted to start with the genetic algorithm as the basic solver.

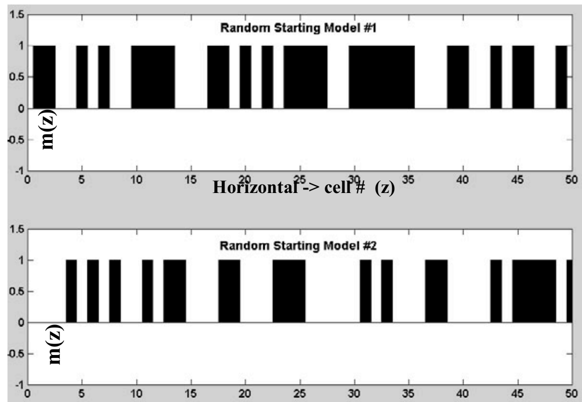

Figure 1. Example of two starting individuals with initialization of random zeros and ones. Each individual represents a potential solution model.

Method

The Genetic Algorithm (GA) This is a programming tool designed for solving a variety of optimization problems. It is a stochastic search technique that mimics natural biological evolution by imposing the principle of 'survival of the fittest' on a population of individuals. The main objective of the GA is to recombine the individuals, with the better-fit individuals having higher probabilities of reproduction, in order to evolve to better solutions.

Individuals The basic unit of the genetic algorithm is the individual. Each individual represents a potential solution to the problem, i.e. a geophysical model. For the binary inverse problem, models are discretized into cells with constant values equal to zero or one. For the GA representation of the model, the individual consists of a series of chromosomes, with each chromosome representing a cell within the model mesh. Therefore, each individual consists of a string of chromosomes, with values of either zero or one.

Initialization The first step in applying the genetic algorithm to gravity inversion is setting up an initial population, which is a community of individuals. Each individual represents a model. For initialization, values are assigned to a model's cells. When prior information is not available, the starting population is randomly initialized, with each individual assigned zeros and ones randomly. Figure 1 displays two examples of random initialization for the GA.

The ability of the genetic algorithm to work with multiple models at one time, through the creation of a population, also allows the user to incorporate prior information. One form of such prior information is the models obtained from previous inversions. It can also be an initial guess produced from other geophysical data such as pre-stack depth migrated seismic data. This is useful in imposing the known top of salt.

Rank, Fitness, and Selection The first step in the evolutionary cycle of the GA is to rank the population. Individuals for our problem are assigned objective values based on eq (1). Lower objective values correspond to higher fitness levels, and therefore better models. Rank is established by ordering the models from highest fitness values to lowest, i.e. best model to worst. Individuals with higher fitness values will have higher probabilities of surviving the evolutionary process as well as passing on their genes to the next generation.

Next we assign selection probabilities defined as the fitness value of an individual divided by the total fitness of the entire population. The highest-ranking model has the highest probability of surviving, while the lowest ranking model has a zero selection probability.

The final step in the selection process is choosing individuals as parents for reproduction. We use Roulette Wheel Selection (Goldberg, 1989) in our algorithm.

Recombination Once individuals have been chosen for recombination based on objective values and selection probabilities, offspring are generated to join the population as the next generation. Selected individuals are paired into parents, and a combination of their chromosomes (model features) are merged to generate offspring. Every new offspring represents a new candidate solution to our problem and it is placed into the next generation.

Mutation After formation of a new set of models, a mutation operator must be applied to protect the population from an irrecoverable loss of potentially useful genetic information during reproduction. In addition, mutation prevents early convergence by introducing new genes into the population, essentially expanding the gene pool and allowing other regions of the solution space to be evaluated. Mutation rates are problem dependent and may be varied according to the performance of the GA. For every child created during recombination, each chromosome has a low probability of being mutated, flipped from zero to one or vice versa.

New Generation The last step in the evolutionary cycle of the genetic algorithm is to evaluate the children and assign objective values. Once these values have been assigned, the children are placed back into the population, often replacing their parents, and creating the next generation of potential solutions. Once the new generation is created, the GA cycle is repeated until convergence.

Numerical Examples

1D Example We first illustrate the binary inversion using a simple mathematical example. The forward mapping is given by the following integral,

$$\begin{aligned} d_j &= \int_0^1 m(z)\cos(2\pi jaz)\exp(-jbz)\,dz \\ &\equiv \int_0^1 m(z)g_j(z)\,dz, \qquad j = 1,\cdots,20 \end{aligned} \tag{3}$$

with $a = b = 0.25$. In eq.(3), $m(z)$ is the model and $g_j(z)$ are the kernels that decay with depth. These kernels are chosen to mimic the decaying kernels seen in many geophysical experiments.

For our numerical test, we use a double boxcar model that is zero everywhere except for two adjacent regions of 1 and -1 respectively, which is shown in Figure 2(b). The simulated data with additive noise are shown as the dots in Figure 2(a). These noisy data, a discretized model mesh of 50 cells, and our objective function (eq.1) are entered into the GA. An initial population of four hundred individuals is initialized. The mutation operator applied during the evolutionary cycle of the GA allows each chromosome within the individuals to mutate with a one in fifty probability. Convergence of the GA is reached at forty generations. The recovered model from the binary inversion is shown in Figure 2(c). The predicted data from this model are shown in Figure 2(a) as the solid line. The binary inversion has performed well in this case.

2.5-D Gravity Problem The second example is a 2.5-D gravity problem. Figure 3(b) displays the true model consisting of a simple block buried in a uniform half-space. The noise-contaminated gravity data taken along a traverse perpendicular to the strike are shown by the dots in Figure 3(a). There are a total of 60 data points. To perform binary inversion, we have divided the model region into 400 rectangular cells (20x20). Each generation of GA has 400 individuals and the algorithm achieves convergence after 150 generations.

The predicted data from the final model are shown in Figure 3(a) as the solid line, which is a smoothed version of the noisy data as expected. The recovered model is shown in Figure 3(c), which compares well with the true model in Figure 3(b).

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Discussion

We have developed a binary formulation for inverting gravity data for subsurface structure that has well-defined density contrasts. The method is designed to overcome the difficulties introduced by nil zones in salt imaging. Initial tests show that the binary formulation provides an effective means to incorporate known density into the inversion. In addition, it can provide a sharp boundary for the salt while maintaining the flexibility of density inversions.

The tradeoff is the increased computational complexity of minimization with binary constraints. We have explored the utility of the genetic algorithm (GA) and shown that it can serve as an effective solver for this problem. Practical applications require more efficient solution methods. Currently, we are exploring other derivative-free methods as well as more efficient implementations of GA by utilizing the special structure of our problem.

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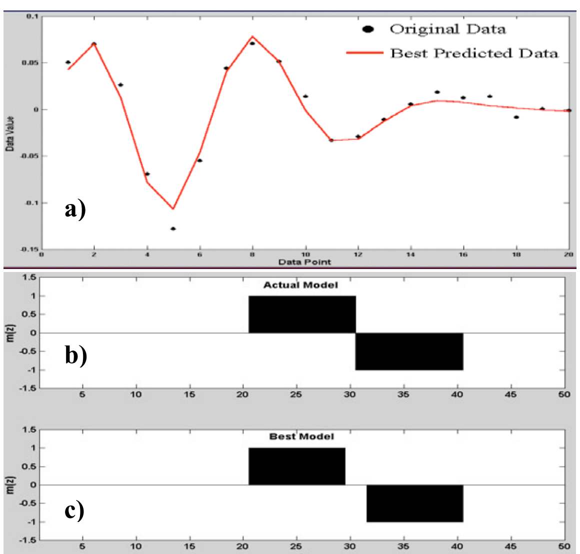

Figure 2. Binary inversion result for the 1D example. Original and predicted data are shown in panel (a), true model in panel (b), and recovered model panel (c).

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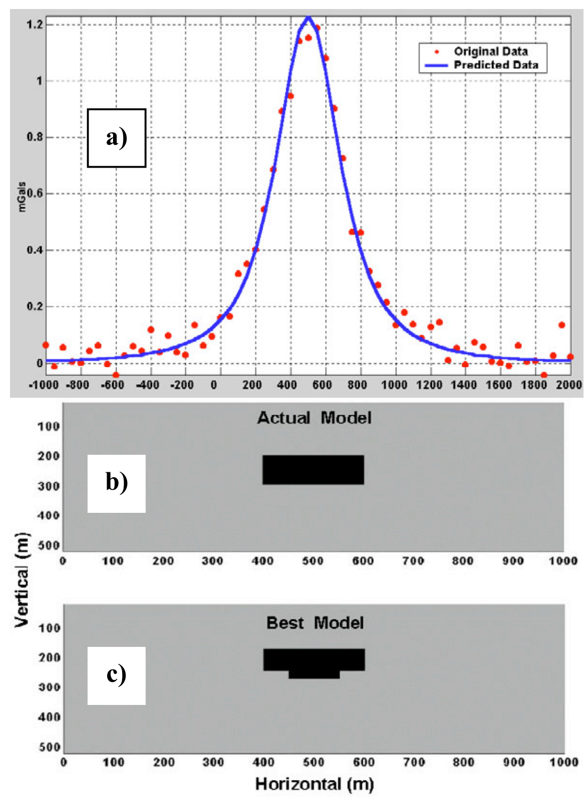

Figure 3. Best model constructed by GA with corresponding data for the 2.5-D gravity problem. (a) shows the observed (dots) and predicted data (solid line), (b) is the true model, and (c) is the constructed model.